Ottawa Team Math Contest

Final Round

Time Limit: 45 minutes.

Details: This is the final round, between the top 2 teams of the competition. In this round, there is only one question. However, this question has 4 preliminary parts. These 4 parts can be solved independently. Upon solving these 4 parts, a team can combine the answers to find the answer to the culminating question.

$$(2 \cdot \texttt{average of individual scores}) + (\texttt{team score})$$

How to win: This round is short answer. The team that arrives at the final answer first wins. If no team has solved the entire problem by the time limit, the team that has solved more parts will win. If both teams have solved an equal number of parts, the team that solved these parts faster will win.

PROBLEMS

Part A (Algebra):

In the following system of equations

$$\begin{cases} x_1^1 + x_2^2 + x_3^3 + \ldots + x_{20}^{20} = 20 \\ x_1 + 2x_2 + 3x_3 + \ldots + 20x_{20} = 210 \end{cases}$$

What is the sum of all x_i of all possible solutions?

Part B (Number Theory):

OTMaC has been on a meteoric rise in popularity in recent years. In fact there are so many participants, it is hard to make groups.

If the contestants are organized in teams of 3, there will be a team short one person. If the contestants are organized in teams of 5, there will be a team short two people. If the contestants are organized in teams of 7, there will be a group with only 2 people. If the contestants are organized in groups of 11, there will be a group of 5 people. What is the minimum number of participants at OTMaC?

Part C (Combinatorics):

Eric, Justin, and Tong Tong are planning to meet up at the bus stop from a time between 12pm and 1pm. Tong Tong is a very patient person so he is willing to wait 30 minutes after he arrives. Justin on the other hand, is quite impatient and will only wait 5 minutes. Eric is in the middle and is willing to wait 10 minutes for the other two. Find the probability that all three meet at the bus stop.

Part D (Geometry):

AB is the diameter of the circle with origin O. C is a point on the tangent line to the circle at A and $AC = AB$. The line segment OC intersects the circle at D and the extension of BD intersects AC at E. Given that $AB = 2$, find the length of AE.