Ottawa Team Math Contest

Team Round

Time Limit: 45 minutes.

Details: This is the second round out of three total rounds. Correct answers score one point. Blank or incorrect answers score zero points. Collaboration is allowed. Calculators are not allowed. Your team score will be equal to the number of questions your team gets correct. The formula used to determine your rank for the last round is

$$(2 \cdot \texttt{average of individual scores}) + (\texttt{team score})$$

Answers: This round is multiple choice and short answer. The first 15 problems are multiple choice and the final 5 are short answer. You DO NOT need to show your work. Write down your choice/final answer to each problem in the answer sheet provided, and do not select several options. Make sure your handwriting is legible - if we can't read it, we won't mark it. Answers should be exact and simplified (e.g. write π instead of 3.14 or $\frac{2\pi}{2}$).

PROBLEMS

1. Of 200 families surveyed, each family own either a dog, a cat, or both. Half of the 150 families who own a dog also own a cat. How many of the people surveyed own a cat?

   a) 65 b) 75 c) 80 d) 100 e) 125

2. The median of a set 2024 consecutive even integers is 2025. What is the sum of all terms in the sequence?

   a) 2024 b) 2026 c) 4 098 600 d) 4 102 650 e) 8 197 200

3. Which of the following numbers is the greatest?

   a) 7^{300} b) 19^{200} c) 50^{100} d) $10^{100} + 20^{100}$ e) $70^{50} + 190^{50}$

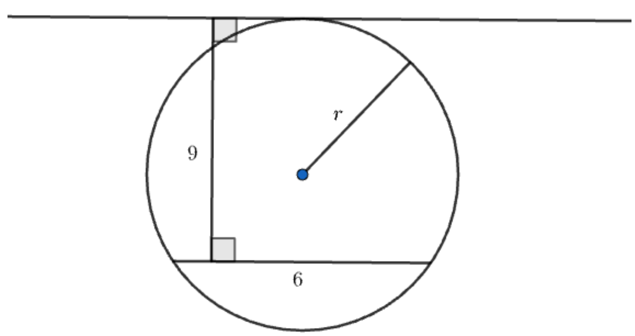
4. Determine the number of divisors of 20!.

   a) 20 b) 210 c) 1152 d) 41040 e) 61560

5. How many equilateral triangles can be formed by connecting the vertices of a cube?

   a) 0 b) 2 c) 4 d) 6 e) 8

6. Find the value of the r in following diagram where r is the radius of the circle,



   a) 2 b) 3 c) 4 d) 5 e) 6

7. Given $f(x) + 2 \cdot f(6 - x) = x$ for all $x \in \mathbb{R}$. Find $f(1)$.

   a) -1 b) 1 c) 2 d) 3 e) 5

8. Which of the following functions is even and has a minimum period of π?

   a) $y = \sin 2x + \cos 2x$ b) $y = \sin^2 x \cos x$ c) $y = \sin^2 x + \cos 2x$
   d) $y = 1 - 2\cos^2 2x$ e) $y = \cos^2 x + \sin 2x$

9. On a 30x30 square grid, Jelly the firefly can move between squares sharing a side (up, down, left, right). Whenever Jelly moves to a different cell, we say Jelly enters that cell, and whenever Jelly moves into a different row/column, we similarly say Jelly enters that row/column. Jelly starts in a corner of the grid and enters each cell exactly once, ending on the corner she started at. What is the smallest integer n such that there exists a path where Jelly enters each row or column no more than n times?

   a) 12 b) 14 c) 15 d) 16 e) 18

10. Evaluate

$$\prod_{i=1}^{4} \cos((20i)^\circ)$$

   a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) $\frac{1}{16}$ e) $\frac{1}{32}$

11. Tony Lu and Long Tu are playing a game. Tony Lu creates a geometric sequence with first term 5 and common ratio $\frac{1}{2}$. Long Tu creates an arithmetic sequence with first term 5 and common difference d. If Tony Lu has the sequence a_1, a_2, a_3, a_4, ... and Long Tu has the sequence b_1, b_2, b_3, b_4, ..., they create a new sequence a_1b_1, a_2b_2, a_3b_3, ...

   Long Tu wins if the sum of the new sequence is greater than or equal to 80. What is the sum of the digits of the minimum value of d that lets Long Tu win?

   a) 3 b) 4 c) 5 d) 6 e) 12

12. Let $P_1P_2 \cdots P_{2024}$ be a 2024-sided convex polygon with no concurrent diagonals. Construct diagonals P_iP_{i+24} where $i = 1, 2, \ldots, 2024$ and $P_{i+24} = P_{i-2000}$ for $2001 \leq i \leq 2024$. How many distinct points of intersection are formed inside the polygon?

   a) 24 288 b) 97 152 c) 93 104 d) 48 576 e) 46 552

13. Determine the difference between the maximal and minimal value of the expression, where $a, b, c \in \mathbb{R}$:

$$\frac{|a+b| + |b+c| + |c+a|}{|a| + |b| + |c|}$$

   a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) 1 d) $\frac{4}{3}$ e) 2

14. Let c be the smallest value such that the reflection of the graph of $y = x^2 - x$ across the line $y = x + c$ passes through $(20, 19)$. c can be expressed in the form $a - \sqrt{b}$, evaluate $a + b$.

   a) 5 b) 13 c) 21 d) 34 e) 58

15. Consider integers $(x, y) \in \mathbb{Z}$ that satisfy the equation:

$$(1 + 2x) \cdot (2 + 2y) \cdot (3x + y) = 12xy$$

Determine the sum of x, y of all pairs of integer solutions for (x, y).

a) -6 b) -2 c) -1 d) 1 e) 3

16. A triangle ABC has $AB = 7$, $BC = 24$, and $\angle ABC = 90°$. A chord BD of the circumcircle of ABC has length 20, where D and B are on opposite sides of AC. Determine the area of the quadrilateral $ABCD$.

17. Given x and y are nonzero complex numbers for which:

$$6x + 5y = |5x| + |5y| = \frac{ix}{y} = z$$

z can be expressed in the form $\frac{a}{b}$, compute the value of $a + b$.

18. Solve the system of equations in positive integers:

$$\begin{cases} ab + cd = 27 \\ ad - bc = 8 \end{cases}$$

19. The year is 2040 and the League of Legends World's Championship is taking place once again. The teams T2 and JENJ are playing in the grand finals and the format is a best of 9. T2 are known for being for being clutch players: the probability of them winning a game is equal to $50\% - 10\% \cdot n$ where n is the difference in the number of wins between T2 and JENJ. Find the probability that JENJ wins.

20. Consider a circle Ω and chord $AB = 16$. Circle ω is tangent to Ω at X and tangent to AB at Y. Let the midpoint of arc $\widehat{AB}$ not including X be M. The length of the tangent from M to ω is 17. Find the area of $\triangle AMB$.

Name: ____________________ **Team ID:** ____________________

P1:	P2:	P3:
P4:	P5:	P6:
P7:	P8:	P9:
P10:	P11:	P12:
P13:	P14:	P15:

P16:
P17:
P18:
P19:
P20: