Ottawa Team Math Contest

Individual Round

Time Limit: 60 minutes.

Details: This is the first round out of three total rounds. Correct answers score one point. Blank or incorrect answers score zero points. Collaboration is not allowed. Calculators are not allowed. Your individual score will be equal to the number of problems you get correct.

Answers: This round is short answer. You DO NOT need to show your work. Write down your final answer to each problem in the answer sheet provided. Make sure your handwriting is legible - if we can't read it, we won't mark it. Answers should be exact and simplified (e.g. write π instead of 3.14 or $\frac{2\pi}{2}$).

PROBLEMS

1. A circle with center O has a point C outside the circle. Two lines pass through C and meet the circle at points A and B so that lines CA and CB are tangent to the circle. The measure of $\angle ACB = 80°$. Point D lies on the major arc $\widehat{AB}$. Find the measure of $\angle ADB$.

2. There are 15 perfect square numbers in the interval $[36a, 49a]$ (inclusive). Find the greatest possible integer value of a that satisfies the condition.

3. Suppose there exist quadratic polynomials $P(x)$, $Q(x)$, and $P(x)+Q(x)$ with vertices of $(4,5)$, $(9,15)$, $(8,80)$ respectively. Find $P(2)+Q(3)$.

4. Consider a trapezoid $ABCD$ with diagonal $BD = 14$, side $CD = 12$, and $\angle BDC = 30°$. Let the diagonals intersect at point O. The ratio of the area of $\triangle COD$ to the area of $\triangle AOB$ is $\frac{9}{16}$. What is the area of $\triangle COD$?

5. Tong Tong drops a circular coin of diameter 4 on a floor that is formed by a tessellation of equilateral triangles of side length 12. What is the probability that the coin lands on at least one line?

6. What is the sum of all possible values of $x+y+z$ where x, y, z are positive integers such that both of the following equations are satisfied?

$$xy + yz = 416$$
$$xz + yz = 41$$

7. A circle with centre O has radius 6. P is a point outside of the circle. A line through P passes through this circle at points A and B such that $\angle AOB = 120$. T is point on major arc $\widehat{AB}$ such that PT is tangent to circle O. Minor arc $\widehat{TB}$ has length 5π. Find the length of PT.

8. Find all integers $n < 1000$ such that there exists an integer m that satisfies $\varphi(n) \mid n \cdot 2^m$. Note that $\varphi(n)$ is the Euler totient function and it counts the number of positive integers less than n that are coprime to n. For example, $\varphi(15) = 8$ because there are 8 numbers less than 15 that are coprime to 15: $\{1,2,4,7,8,11,13,14\}$.

9. Find the minimum value of the expression: $\sqrt{2}a^3 + \frac{3}{ab-b^2}$, given $a > b > 0$.

10. Jayce loves hexagons and commonly uses them in his hextech engineering. One day, on the rift, he finds himself on a hexagon on vertex A. He can jump from one vertex of the hexagon to an adjacent one. If he lands on the opposite vertex D, he stops. Find the number of 19-jump paths he can take to end up at D.

Name: ______________________ **Team ID:** ______________________

P1:

P2:

P3:

P4:

P5:

P6:

P7:

P8:

P9:

P10: