Ottawa Team Math Contest

Team Round

Time Limit: 1 hour

Instructions:

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Calculating devices are ***not*** allowed.
4. Diagrams are ***not*** drawn to scale. They are intended as aids only.
5. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. The correct answer to each question in Part C can be any real number.
6. Scoring:
    a. Each correct answer is worth 3 in Part A, 4 in Part B, and 6 in Part C.
    b. There is no penalty for an incorrect answer.
    c. To discourage guessing, each unanswered problem in Part A and B is awarded 1 point, and a maximum of 5 points may be awarded for omitting problems.
7. Collaboration within the team is allowed. Collaboration between teams is ***not*** allowed.

Answers:

You DO NOT need to show your work. Write down your choice/final answer to each problem in the answer sheet provided, and do not select several options. Make sure your handwriting is legible - if we can't read it, we won't mark it. Answers should be exact and simplified (e.g. write π instead of 3.14 or $\frac{2\pi}{2}$).

Problems

Part A: Each correct answer is worth 3.

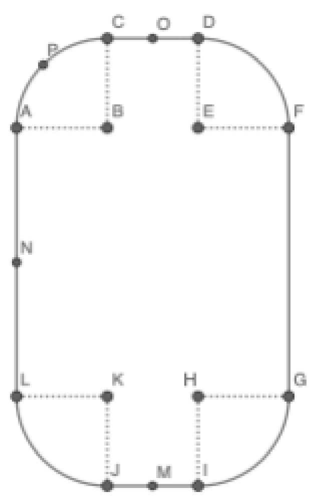
1. What is the last digit of the product of the first 2025 odd prime numbers?

   A) 1 B) 3 C) 5 D) 7 E) 9

2. Suppose Eric adds up all the digits of the positive integer 3^{2025}, getting the positive integer n_1. Then, he adds up all the digits of n_1 to get n_2. He keeps repeating this process until he ends up with a one-digit number. What is that number?

   A) 1 B) 3 C) 5 D) 6 E) 9

3. A hockey rink (as shown) is defined by a 12-sided polygon ABCDEFGHIJKL where angles measures of B, E, H, K are 270º, AL = FG = 3, the remaining lengths are 1, and four quarter circles join the corners to complete the shape. Let a puck rebound off the perimeter of ACDFGIJL such that its incident angle is equivalent to its rebounded angle. Given the points M, N, O, P, find what point puck P just rebounded off of, given it started at point M in the direction of point N and has already rebounded 2025 times.



   A) M B) N C) O D) P E) None of the above

4. What is the remainder when $6^6 + 7^7 + 8^8$ is divided by 10?

A) 5 B) 6 C) 7 D) 8 E) 9

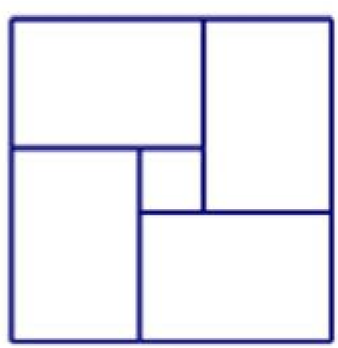
5. BattleCatsBoy is trying to build a deck of 8 cards, and each card in this game ranges from 1 to 9 elixirs, inclusive (For those who play Clash Royale, “The Mirror” is not considered as a card here). The application calculates the deck’s average elixir cost by summing up all individual card’s elixir costs and dividing it by 8 - rounding it to the nearest 1 decimal place. Given that there are exactly 6 one-elixir cards in the game, and unlimited number of other cards, what is the average elixir cost of the 13th lowest elixir deck one can build; in other words, what is the 13th lowest possible average elixir cost in the game?

A) 2.4 B) 2.5 C) 2.6 D) 2.7 E) 2.8

6. There are 5 lines in a plane, and 3 of these lines are concurrent to each other (i.e. they intersect at one point), and the rest are not. None of the lines are parallel to each other. How many points of intersections are there?

A) 6 B) 8 C) 10 D) 12 E) 14

7. A large square is divided into 4 congruent rectangles and a smaller square as shown. Given that each rectangle has area 35 and the small square has area 4, find the positive difference between the length and the width of each rectangle.



A) 1 B) 2 C) 3 D) 4 E) 5

8. Three couples each send text messages on a fixed schedule between 8:00 AM and 9:00 AM—which we treat as minute 0 through minute 60:
   - Karan & Vickie: First text at minute 0, then every 6 minutes (0,6,12…)
   - Matthew & Emily: First text at minute 1, then every 7 minutes
   - Eric & Juliann: First text at minute 3, then every 11 minutes

   A text event is a minute in which at least one couple sends a text. How many total text events occur between minutes 0 and 60 (inclusive)?

   A) 21 B) 22 C) 23 D) 24 E) 25

9. $x + y + z = 15$, $xy + yz + xz = 74$. Compute $x^2 + y^2 + z^2$.

   A) 57 B) 67 C) 77 D) 87 E) 97

10. Define a 45º or 30º measure angle as a "special angle". How many unique **scalene** triangles are there such that all 3 of the angles have a measure of integer degree and have at least one special angle?

    A) 138 B) 140 C) 141 D) 284 E) 285

Part B: Each correct answer is worth 4.

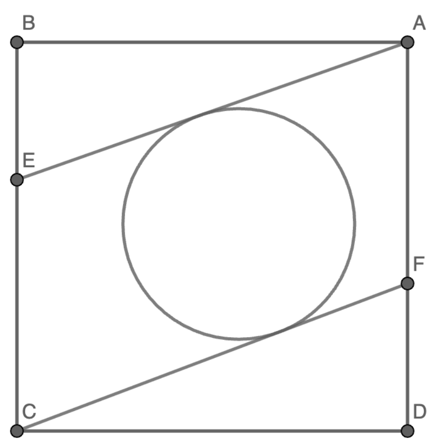
11. The nine squares of a 3×3 grid are to be colored red, white, and blue, so that no row or column contains squares of the same color. How many different patterns can be made?

A) 4 B) 6 C) 9 D) 12 E) 18

12. Let S be the value of the following expression:
$(1 + \frac{2}{1})(1 + \frac{2}{2})(1 + \frac{2}{3})(1 + \frac{2}{4}) \cdots (1 + \frac{2}{2024})(1 + \frac{2}{2025})$
What is the sum of all digits of S?

A) 19 B) 20 C) 21 D) 22 E) 23

13. Let ABCD be a square with area 10. Point E is on line segment BC such that CE = 2BE; point F is on line segment AD such that AF = 2DF. A circle is drawn so that it is tangent to AE and CF, what is the area of that circle?



A) $\frac{2}{3}\sqrt{10}\pi$ B) $\frac{3}{\sqrt{10}}\pi$ C) π D) $\frac{\sqrt{10}}{3}\pi$ E) $\frac{10}{9}\pi$

14. Some positive integers sum up to 11 and their maximum possible product is P; some other positive integers have a product of 80 and their minimum possible sum is S. What is P - S?

A) 17 B) 32 C) 35 D) 41 E) 43

15. There are unique integers $a_2,\ a_3,\ a_4,\ a_5$ such that $\frac{4}{5} = \frac{a_2}{2!} + \frac{a_3}{3!} + \frac{a_4}{4!} + \frac{a_5}{5!}$ subject to $0 \leq a_i < i$ for $i = 2, 3, 4, 5$. Compute $a_2 + a_3 + a_4 + a_5$

A) 6 B) 7 C) 8 D) 9 E) 10

16. Let ABCD be a trapezoid where AB || CD and AB > CD. Let the angle bisector of B intersect AD at E such that E lies on the circumcircle of $\triangle$ABC. Given that $\angle$ABE = 15º, $\angle$ADC = 90º, and DC = 2. The length of BE can be represented as $a(\sqrt{b} + \sqrt{c})$ for some positive integers a, b, and c, where b and c do not have any perfect square divisors other than 1.

What is a + b + c?

A) 8 B) 9 C) 10 D) 11 E) 12

17. Rex wants to play PUBG with Karan, and they decided to meet at McDonald's this morning. Rex really wants to play PUBG but sleeps too much sometimes, so he can arrive any time at McDonald's between 9am and 11am and he can stay for at most 2 hours but he must leave no later than 12pm. Karan says he has to do robotics, so he will leave immediately if he doesn't see Rex at McDonald's the moment he arrives, and Karan can arrive any time between 9am and 12pm. If the probability of Rex successfully meeting Karan can be expressed as $\frac{m}{n}$ such that m and n are coprime, what is m + n?

A) 7 B) 9 C) 13 D) 17 E) 19

18. Let I be the intersection of two chords AB and CD, where BI > DI. Take D’ on BI such that DI = DD’. Given AI = 2, CI = 3, D’I = $2\sqrt{6} - 2\sqrt{2}$, and $\angle$DD’I = 75º, Find BI.

A) 5 B) 6 C) 7 D) 8 E) 9

19. If a Baby Dragon weighs 100kg, an Ice Golem weighs 85kg, a Hog weighs 70kg, and there are exactly X Baby Dragons, Y ice golems, and Z Hogs that weigh a total of 2025kg (X, Y, Z are nonnegative integers), how many possible triples of (X, Y, Z) are there?

A) 17 B) 18 C) 19 D) 20 E) 21

20. The only non-integer rational solution to the following polynomial $9x^6 + 12x^5 - 2x^4 - x^3 + 3x^2 - x$ can be represented as a/b, where a and b are coprime integers. What is |100a - b|?

A) 97 B) 197 C) 297 D) 397 E) 497

Note: There was a small mistake in problem 20 during the contest, so the points were awarded to all the teams. However, it is now fixed in this version.

Part C: Each correct answer is worth 6.

21. Triangle ABC has a right angle at B and contains a point P such that PA = 10, PB = 6, and $\angle APB = \angle BPC = \angle CPA$. Find the length of line segment PC.

22. If integers a, b, c, d form a geometric sequence such that $0 < a < b < c < d < 1000$, how many possible quadruples of (a, b, c, d) are there?

23. A number set S contains 7 elements: $S = \{2, 3, 4, 6, 9, 12, 18\}$. If the product of a set is the product of all the elements in it, how many subsets of S (including S and empty set) has a product that is a multiple of 36?

24. What is the sum of all positive divisors of 2025^2 that has a last digit of 1 OR 5?

25. In Clash Royale's Path of Legends, players gain 1 step for a win and lose 1 step for a loss, unless they are on a Golden Step, which cannot be lost. A Golden Step is earned after winning 3 games (not necessarily consecutively) since the last Golden Step. Once earned, it remains permanently. Rex starts at step 0 (a Golden Step) and needs to reach step 10 to enter Challenger III. <u>Given that Rex has a 80% probability of winning each game, what is the probability that he reaches Challenger III within 12 games? Round to 2 significant digits.</u>

    Useful Fact: $0.8^{10} \approx 0.1074$

    Example Scenario:

    - ➢ Rex starts at step 0 (a Golden Step).
    - ➢ He loses his first 10 games but stays at step 0 because step 0 is a golden step.
    - ➢ He then wins 2 games (reaching step 2) but loses the next game (dropping to step 1).
    - ➢ Winning another game moves him back to step 2, and since he has now won 3 games total since step 0, step 2 becomes a Golden Step.
    - ➢ From here, he can never drop below step 2, no matter how many times he loses.
    - ➢ The process repeats: every time Rex accumulates 3 wins after the previous Golden Step, he earns a new one.