

# Welcome to the Knockout Round!

OTMaC 2025

# Rules

The knockout round is a single elimination tournament bracket where qualified teams will face off against each other in team versus team math speed matches to determine the winner of OTMaC 2025!

Each team versus team match will consist of a maximum of 5 rounds. At the start of each round, your team will choose one member who has not yet gone in the match to go up against your opponent's chosen member. The final match will be best 2 out of 3.

In order to keep your chosen member unknown to the other team, the **entire team order** for the first four problems will be submitted on paper slips to Rex the Rigid Referee.

For example: P1 - Rex, P2 - Eric, P3 - Matthew, P4 - Karan

# Rules P2

After contestants are chosen, teams will take turns sending contestants who will be presented with the same math problem to solve in under 90 seconds (1m 30s).

The first four problems will be ordered in difficulty, where the first one will be the easiest and the fourth will be the most difficult, so choose your team order wisely!

For the first four problems it **does not** matter if you are faster or slower than your opponent. As long as you finish the problem under the time limit, your team will be awarded 1 point.

To prevent guessing during the first four problems, contestants can only submit a maximum of 2 submissions before disqualification. To submit, box an answer on your scrap and raise your hand, as long as your answer is boxed before the 90 seconds is over, your solution will count.

# Rules P3

If the score is tied by the end of the fourth problem, the match will enter sudden death, where each team sends all four members in to solve a (much) harder problem. There is no time limit - the fastest team to solve the problem will move on to the next round!

To prevent guessing, incorrect answers in the final round will result in a 15 second time penalty where teams cannot submit another solution.

Additional note: In general, problems will be harder in the later rounds.

Round 1

## Round 1 Problem 1

Calculate:  $10 + 11 + 12 + \dots + 19 + 20$

## Round 1 Problem 2

What is the unit digit of the sum of the first 98 positive integers?

## Round 1 Problem 3

The mean of the set of numbers, (5, 10, 13, 7, 21, x, y) is 15.

Furthermore,  $x - y = 3$ .

Find x.



## Round 1 Problem 4

When Clash Royale was created in 2016, Baby Dragon was 3 years old, and this year (2025) Electro Dragon is 3 times older than Baby Dragon. How old was Electro Dragon in 2016?

## Round 1 Team Sudden Death

What is the minimum number of intersections between the 4 congruent circles of area  $9\pi$ , where the circles are conformed to be placed inside a square with area 100?

# Round 2

## Round 2 Problem 1

Calculate:  $1^3 + 2^3 + 3^3 + 4^3 + 5^3$

## Round 2 Problem 2

In an arithmetic sequence  $a_1, a_2, a_3, \dots$ , the term  $a_3=7$  and  $a_{10} = 49$ . What is the value of  $a_7$ ?

## Round 2 Problem 3

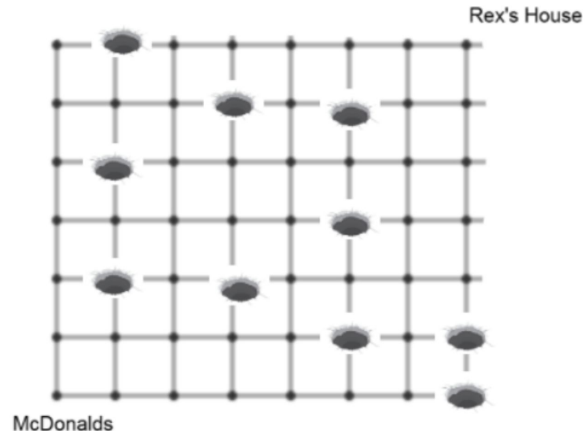
An infinite geometric series with a positive common ratio  $r$  has a total sum of 5. The first element in the series has a value of 3. Find  $r$ .

## Round 2 Problem 4

What is the value of  $|10^2-13^2| + |13^2-12^2| + |12^2-11^2| + |11^2-10^2|$ ?

## Round 2 Team Sudden Death

After a long day of playing PUBG with Karan, Rex needs to get back home. Rex lives in McDonaldsville, where all streets form a grid pattern, with each street spaced 1 km apart from the nearest parallel streets; however, some intersections have been rocketed by Mr. Gibson and cannot be crossed! How many different ways can Rex get home such that he avoids the craters while taking the shortest possible path?





Round 3

## Round 3 Problem 1

What is the maximum number of intersection points of an equilateral triangle and a square?

## Round 3 Problem 2

As soon as BattleCatsBoy starts playing Clash Royale again, he drops his trophy number to half each day he plays. Given that he started at 9000, how many trophies would he have left after playing for 3 days?

## Round 3 Problem 3

Everyday, Rex plays slowroads.io for a fixed amount of time. If Rex's average speed was 2.5 km/min yesterday, and he drove a total of 200 km, then how far in km would Rex drive today if he increased the speed of yesterday by 30 km/h today.

## Round 3 Problem 4

The number of possible arrays of 4 positive integers such that each element has even digits and no element is greater than 100 can be expressed as  $x^2$ ? What is  $|x|$ ?

## Round 3 Sudden Death

Consider a prime number  $p$ . Let  $r(p)$  be the number obtained by reversing the digits of  $p$ . For example,  $r(103) = 301$ .

$p$  is considered reversible if both  $p$  and  $r(p)$  are prime. Find the smallest reversible  $p$  greater than 400.

# Round 4

## Round 4 Problem 1

Find  $x$  such that:

$$32^x = 64^{10}$$



## Round 4 Problem 2

There are 100 STEM students in Merivale High School. While 20 of them are not in any clubs, the others are all in at least one STEM club. If there are 50 students in Math Club and 15 in both the Math Club and the Science Club, how many students are in the Science Club? (You may assume Math and Science Club are the only STEM clubs in MHS)

## Round 4 Problem 3

Baby Dragon puts a \$5, a \$10, a \$20, a \$50, a \$100 bill in 5 of his eggs where each egg has one bill, and he takes 2 of the eggs with him today to shop. What is the probability that he has more than \$40 to shop today?

## Round 4 Problem 4

$$x^2 + y^2 = 416$$

$$y = -5x$$

What is  $|y - x|$ ?

## Round 4 Sudden Death

Find the smallest positive  $x$  (in radians) such that:

$$13\sin x - 14\cos^2 x = -4$$

Round 5

## Round 5 Problem 1

What is the remainder when 2025 is divided by 23?

## Round 5 Problem 2

Solve the following system of equations for positive solutions of  $(x, y)$ :

$$x = 17y$$

$$x^2 + y^2 = 290$$

## Round 5 Problem 3

A sequence is defined by  $a_6 = 95$  and  $a_{n+1} = 2a_n + 1$ . Determine  $a_1$ .



## Round 5 Problem 4

What is the product of all solutions in  $x^3 - 6x^2 + 11x - 6$ ?

## Round 5 Sudden Death

$$x + y + z = 3$$

$$x^2 + y^2 + z^2 = 17$$

$$y \cdot z = -6$$

Find all possible values of  $x$

Round 6

## Round 6 Problem 1

What is the sum of the first 11 prime numbers?

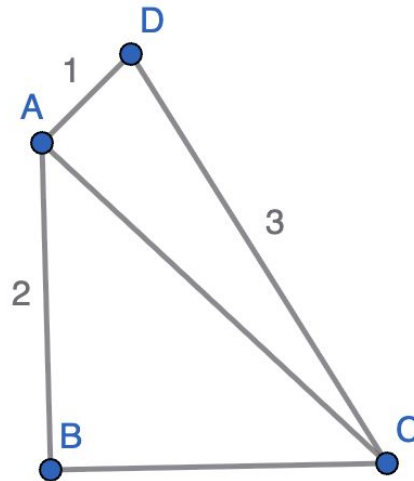
## Round 6 Problem 2

What is the 17th positive odd integer not divisible by 3?

## Round 6 Problem 3

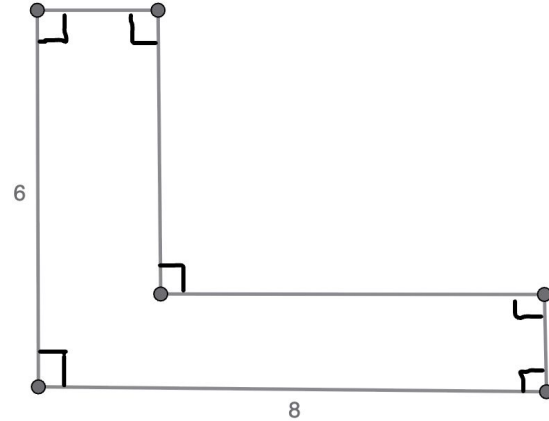
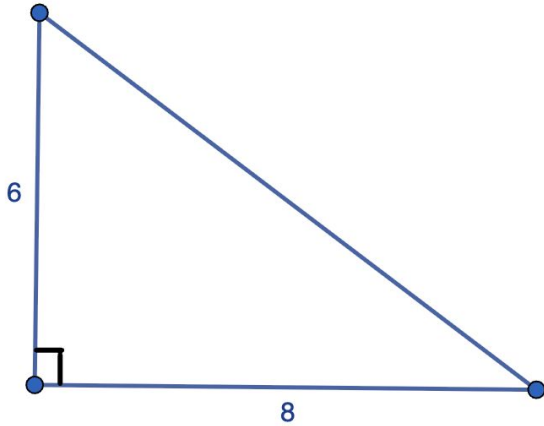
In the following diagram,  $\angle ABC = 90^\circ$ ,  $\angle ACB = 45^\circ$ ,  $AB = 2$ ,  $AD = 1$ ,  $CD = 3$ .

Find the measure of  $\angle BAD$  in degrees.



## Round 6 Problem 4

Find the positive difference between the perimeter of the following two shapes.



## Round 6 Sudden Death

Find the smallest positive  $x$  (in radians) such that

$$\sin 2x - 1 = \frac{1 - \sin 2x}{\sin x}$$



# Round 7

## Round 7 Problem 1

How many composite numbers are there between 100 and 125, inclusive.

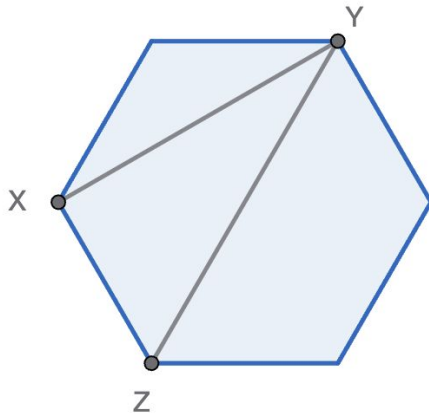
## Round 7 Problem 2

Evaluate to the nearest integer

$$e^3 + \pi$$

## Round 7 Problem 3

In the regular hexagon below, what is the length of  $XY$  given that  $YZ = 4$ ?



## Round 7 Problem 4

How many distinct real roots does the following polynomial have?

$$x^4 - 8x^3 + 23x^2 - 28x + 12$$

## Round 7 Sudden Death

A deck of cards consists of 52 cards without jokers. What is the probability of randomly selecting, without replacement, a spade, followed by a face card in that order?

Round 8

## Round 8 Problem 1

Calculate:

$$2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^3 + 2^0$$



## Round 8 Problem 2

Karan needs to go home from playing PUBG at McDonalds, but he lives in an igloo far away in the arctic, 3000 km away! In order for Karan to get home, he needs to fly on a plane traveling 500 km/h for 1700 km, wait an hour at the airport to fly on another plane traveling 750 km/h for 1200 km, then drive 100km/h for the rest of the distance. What time will Karan get back home if he leaves at 9 pm?

## Round 8 Problem 3

$x$  and  $y$  are positive integers greater than 1 such that

$$x^y = 4096$$

What is the sum of all possible values of  $x + y$ ?

## Round 8 Problem 4

A cube of side length  $2\sqrt{3}$  is inscribed within a sphere. What is the volume of the sphere?

## Round 8 Sudden Death

A standard, six-sided die is rolled exactly 3 times. What is the probability that the 3 rolls were in strictly increasing order? For example, if the first roll is 1, the second roll could be 2, and the third could be 3.

Round 9

## Round 9 Problem 1

The following set of numbers has a mean of 10, what is  $x$ ?

1, 1, 2, 3, 5, 8,  $x$ , 21, 34

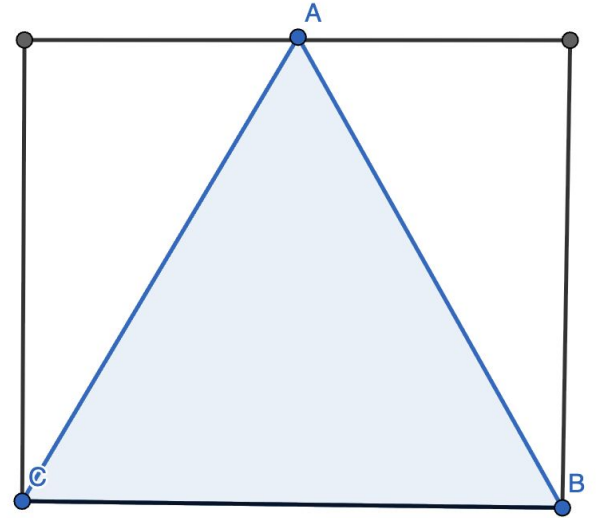
## Round 9 Problem 2

Hog rider weighs 100kg and has a Body Mass Index (BMI) of 25, goblin weighs 30kg and has a Body Mass Index of 15. How many centimeters is a hog rider taller than a goblin? (You may round your answer to the nearest integer)

Body Mass Index formula:  $BMI = \frac{\text{mass in kg}}{(\text{height in m})^2}$

## Round 9 Problem 3

In the following diagram, equilateral  $ABC$  is inscribed in a rectangle with an area of  $4\sqrt{3}$ . What is the value of the side length of  $ABC$ ?

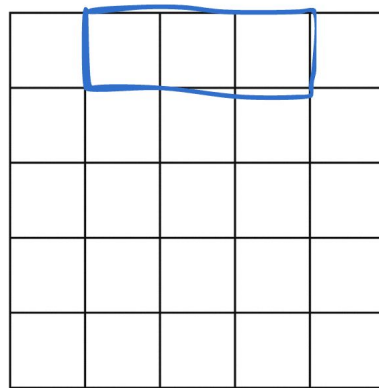
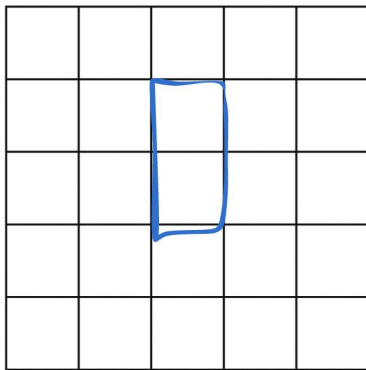
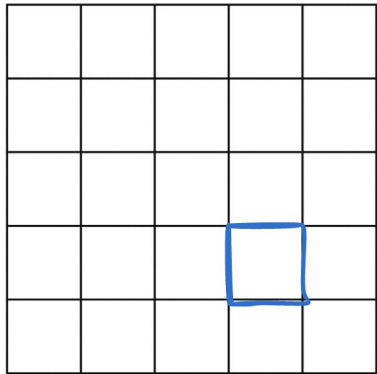




## Round 9 Problem 4

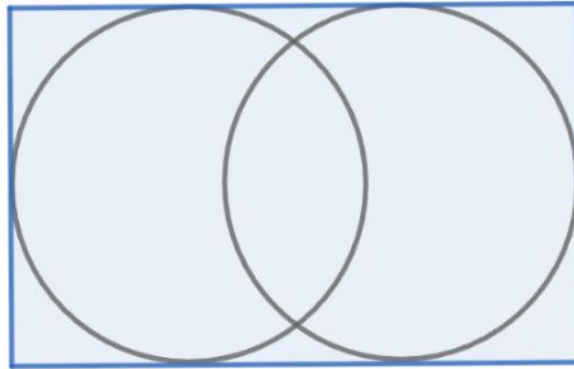
In a  $5 \times 5$  grid of unit squares, we form rectangles by choosing grid lines for the boundaries. Among all such possible rectangles, how many have an odd integer area?

Example rectangles cut from the grid:



## Round 9 Sudden Death

A rectangle with a length of 6 and width of 4 has two inscribed circles, each touching three sides of the rectangle. What is the **overlapping** area of the two circles?



# Round 10 - 3rd Place Match

## Round 10 Problem 1

The number 12240 can be prime factorized into  $a^b \cdot c^d \cdot e^f \cdot g^h$ . What is  $a + b + c + d + e + f + g + h$ ?

## Round 10 Problem 2

Gibson's hair rage-grows by 0.5 cm when Rex is late, and Rex always has a 50% probability of being late. If Gibson is completely bald on a Sunday, and after a full week of 5 days, what is the probability that his hair is 2 cm or longer? (You may assume Rex is a responsible student who attends math class everyday)

## Round 10 Problem 3

Solve for all real values of x:

$$\log_3(x^2 + 4x) - \log_3(x + 4) = \frac{1}{2}\log_5 25$$

## Round 10 Problem 4

How many unique combinations of 4 slices in a circle with an area of  $38\pi$  are there such that each slice has an area of  $k\pi$  where  $k$  is an integer and a minimum area of  $8\pi$ ?

## Round 10 Sudden Death

Find the smallest positive  $x$  (in radians) such that:

$$\sin(2x) = \sin(2x)(\sin(x)) + 2(\cos^3 x)$$



# Round 11 - 3rd Place Match

## Round 11 Problem 1

Matthew rolls two regular dice, what is the probability that their sum is at least 10?

## Round 11 Problem 2

Find  $|a - b|$  such that:

$$a^2 - b^2 = 5$$

$$ab = 6$$

## Round 11 Problem 3

In the  $xy$ -plane, the segments with endpoints  $(8,0)$  and  $(0,6)$  is a diameter of a circle. If the point  $(x, 8)$  is on the circle, what is  $x$ ?

## Round 11 Problem 4

There is a value of  $k$  such that for all  $b < k$ ,  $(3-b)x^2 + (2-2b)x + (5-b)$  has no real solutions. Find the value of  $k$ .

## Round 11 Sudden Death

In a bag of marbles there is 1 red marble, 2 green marbles, and 2 blue marbles. One by one, they are chosen without replacement from the bag. Given the condition that no two same colour marbles were chosen consecutively, what is the probability that the 4th marble chosen was a blue marble?

# Round 12 - Finals

## Round 12 Problem 1

Solve for x:

$$\frac{1}{x} + \frac{1}{2x} + \frac{1}{5x} = 17$$



## Round 12 Problem 2

What is the coefficient of  $x^6$  in this polynomial?

$$(x+1)(x^2+2)(x^3+3)(x^4+4)$$

## Round 12 Problem 3

Suppose there are between 80 to 102 students participating in OTMaC 2026, and the 4 organizers want to split the students into multiple groups of equal number of students. However, they realize that it doesn't work, so all 4 of them join the students, and try to split into groups again, but it still doesn't work. How many student participants are there exactly?

## Round 12 Problem 4

Solve for all values of  $x$

$$13 - 3\log_8(x) = \frac{14}{\log_8(x)}$$

## Round 12 Sudden Death

Two distinct lines are drawn from one corner of a regular hexagon to another such that the two corners are not adjacent (the lines aren't just the edges of the hexagon itself). The lines are considered intersecting if they intersect strictly inside the hexagon. What is the probability that the two lines intersect?